



# Measuring the Neutrino Mass Hierarchy: Atmospheric Neutrinos

Dan Dwyer  
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# Outline

## **Atmospheric Neutrinos:**

Signature of the neutrino mass hierarchy

Measurement Approach

Impact of existing oscillation uncertainties

Challenges

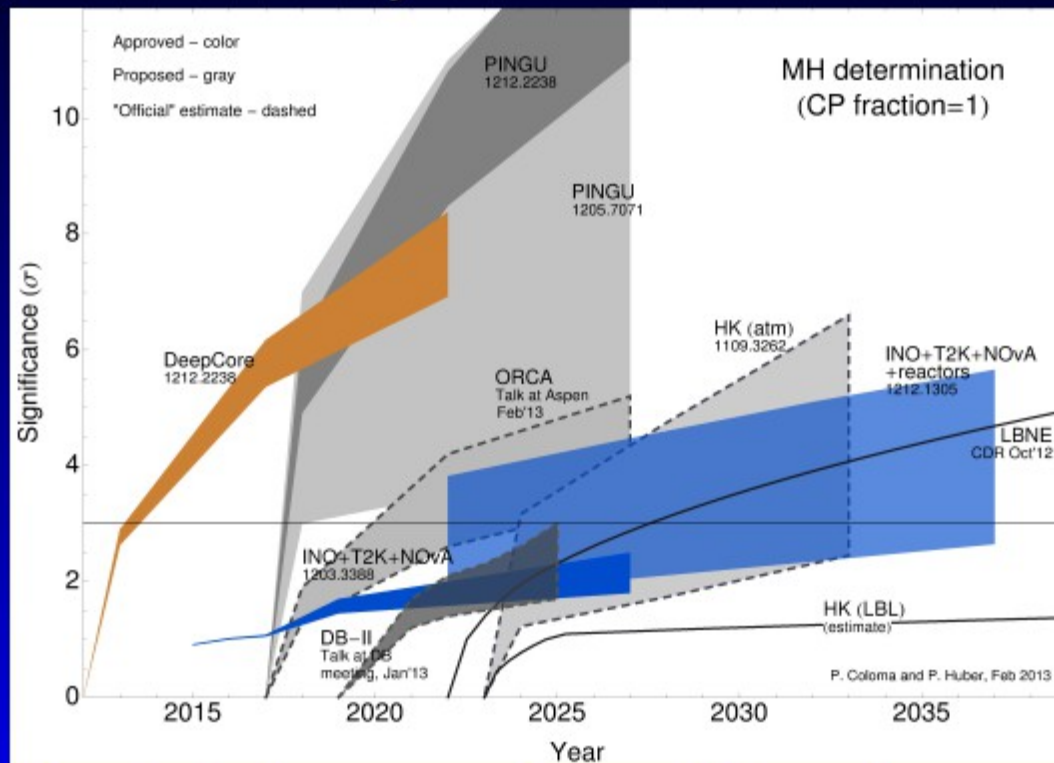
Schedule and Prospects

Closing Statements

# Significant Interest

## New ideas for mass hierarchy

### Literature survey



The dashed ones are from collaborations – phenomenological studies are driving the field.

Patrick Huber, SLAC Intensity Frontier Pre-Snowmass Meeting

# Hierarchy and Oscillation

## Matter potential:

$$V = \sqrt{2} G_F \underline{N_e}$$

$e^-$  density

$$\mathcal{H}_m = H_m + V_m$$

$$i \frac{d}{dt} \psi_f(t) = \mathcal{H}_f \psi_f(t)$$

## Two-flavor:

$$\mathcal{H}_f = \frac{\Delta m_{\odot}^2}{4E} \begin{bmatrix} -\cos 2\theta_{\odot} & \sin 2\theta_{\odot} \\ \sin 2\theta_{\odot} & \cos 2\theta_{\odot} \end{bmatrix} + \begin{bmatrix} V(r) & 0 \\ 0 & 0 \end{bmatrix}$$

hierarchy                      matter effect

## Three-flavor:

$$\mathcal{H}_m = H_m + U^{-1} V_f U.$$

$$\mathcal{H}_m = \begin{pmatrix} E_1 + AU_{e1}^2 & AU_{e1}U_{e2} & AU_{e1}U_{e3} \\ AU_{e2}U_{e1} & E_2 + AU_{e2}^2 & AU_{e2}U_{e3} \\ AU_{e3}U_{e1} & AU_{e3}U_{e2} & E_3 + AU_{e3}^2 \end{pmatrix}$$

Follows arXiv:hep-ph/9910546

## Oscillation solution....

$$e^{-i\mathcal{H}_m L} = \phi e^{-iLT} = -i\phi \frac{U^\dagger}{D} \left\{ [e^{-iL\lambda_1} (\lambda_2^2 \lambda_3 - \lambda_1 \lambda_2^2) + e^{-iL\lambda_2} (\lambda_1 \lambda_3^2 - \lambda_2^2 \lambda_3) + e^{-iL\lambda_3} (\lambda_1^2 \lambda_2 - \lambda_1 \lambda_2^2)] I + [e^{-iL\lambda_1} (\lambda_2^2 - \lambda_1^2) + e^{-iL\lambda_2} (\lambda_3^2 - \lambda_1^2) + e^{-iL\lambda_3} (\lambda_3^2 - \lambda_2^2)] T + [e^{-iL\lambda_1} (\lambda_3 - \lambda_2) + e^{-iL\lambda_2} (\lambda_3 - \lambda_1) + e^{-iL\lambda_3} (\lambda_1 - \lambda_2)] T^2 \right\} \quad (42)$$

Inserting the expression for  $D$  into Eq. (42) gives

$$e^{-i\mathcal{H}_m L} = \frac{1}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \phi e^{-iL\lambda_1} [\lambda_2 \lambda_3 I - (\lambda_2 + \lambda_3)T + T^2] + \frac{1}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \phi e^{-iL\lambda_2} [\lambda_1 \lambda_3 I - (\lambda_1 + \lambda_3)T + T^2] + \frac{1}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \phi e^{-iL\lambda_3} [\lambda_1 \lambda_2 I - (\lambda_1 + \lambda_2)T + T^2]. \quad (43)$$

Using the various relations for the eigenvalues finally yields

$$e^{-i\mathcal{H}_m L} = \phi e^{-iL\lambda_1} \frac{(\lambda_1^2 + c_1)I + \lambda_1 T + T^2}{3\lambda_1^2 + c_1} + \phi e^{-iL\lambda_2} \frac{(\lambda_2^2 + c_1)I + \lambda_2 T + T^2}{3\lambda_2^2 + c_1} + \phi e^{-iL\lambda_3} \frac{(\lambda_3^2 + c_1)I + \lambda_3 T + T^2}{3\lambda_3^2 + c_1}, \quad (44)$$

which can be written as

$$U_m(L) = e^{-i\mathcal{H}_m L} = \phi \sum_{n=1}^3 e^{-iL\lambda_n} \frac{1}{3\lambda_n^2 + c_1} [(\lambda_n^2 + c_1)I + \lambda_n T + T^2]. \quad (45)$$

The evolution operator for the neutrinos in the flavor basis is thus given by

$$U_f(L) = e^{-i\mathcal{H}_f L} = U e^{-i\mathcal{H}_m L} U^{-1} = \phi \sum_{n=1}^3 e^{-iL\lambda_n} \frac{1}{3\lambda_n^2 + c_1} [(\lambda_n^2 + c_1)I + \lambda_n T + T^2], \quad (46)$$

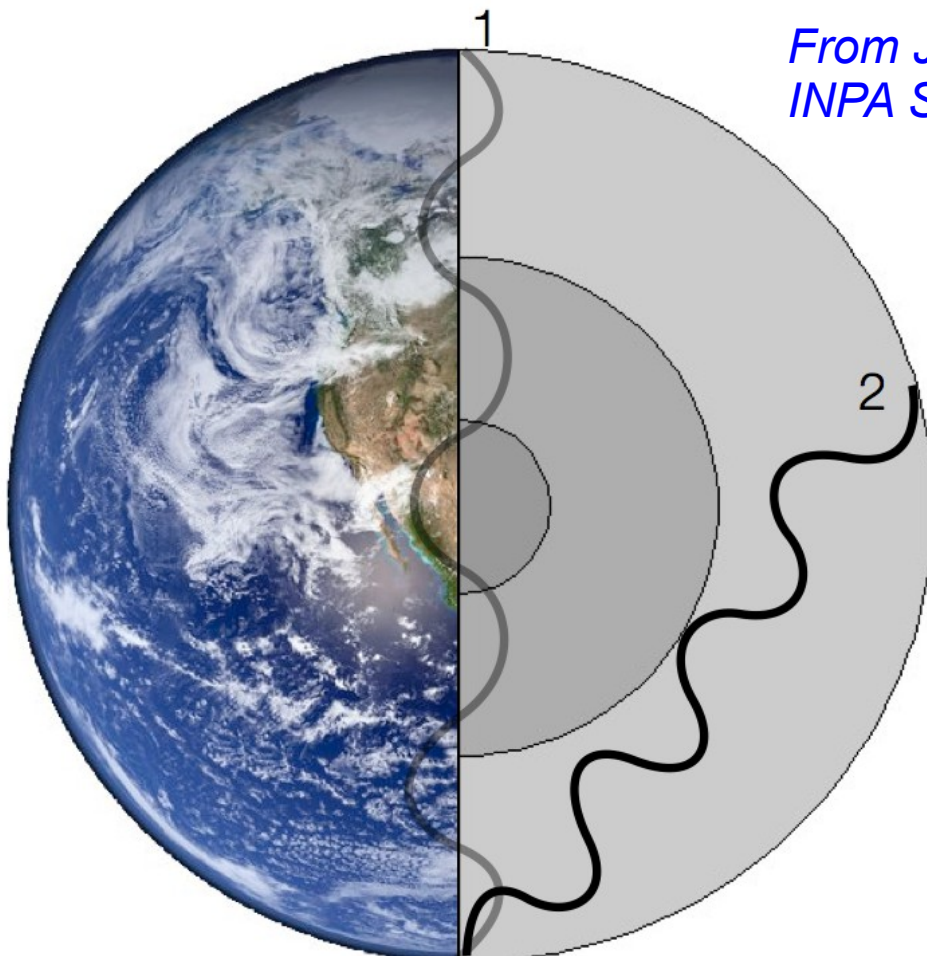
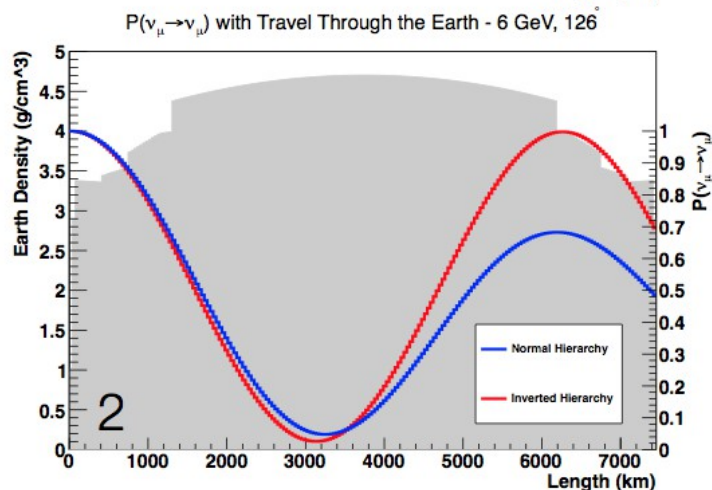
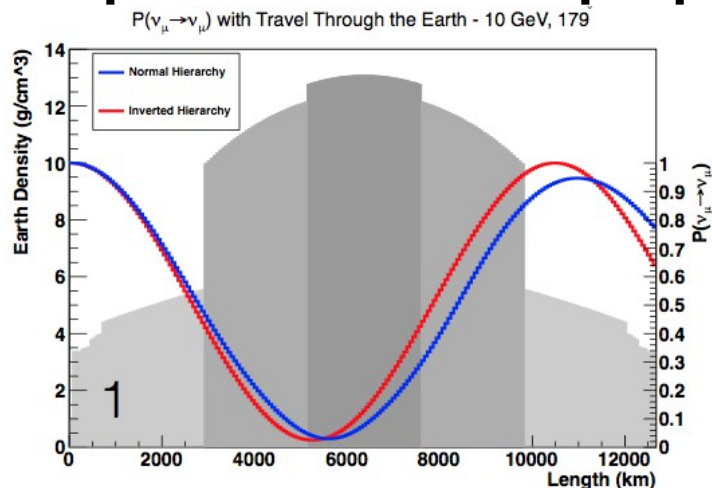
where  $\tilde{T} \equiv U U^{-1}$ . Equation (46) is our final expression for  $U_f(L)$ .

Let us pause for a moment to contemplate Eqs. (45) and (46). Since  $\mathcal{H}_f = U \mathcal{H}_m U^{-1}$ , it is clear that  $\tilde{T} = \mathcal{H}_f - (\text{tr } \mathcal{H}_f)I/3 = \mathcal{H}_f - (\text{tr } \mathcal{H}_m)I/3$  due to the invariance of the trace under transformation of  $U$ . In fact, the characteristic equation is also invariant under  $U$  and therefore so are the coefficients  $c_0, c_1, c_2$ , and the eigenvalues  $\lambda_1, \lambda_2, \lambda_3$ . However, the expression for  $\mathcal{H}_f$  is much more complicated than that for  $\mathcal{H}_m$ , which is the reason why we work with  $\mathcal{H}_m$  instead of  $\mathcal{H}_f$ .

The formula (46) is our main result for the evolution operator. It expresses the time (or  $L$ ) evolution directly in terms of the mass squared differences and the vacuum mixing angles without introducing any auxiliary matter mixing angles.

# Earth Oscillation

Atmospheric neutrinos propagate across varying earth density profile.



*From J. Koskinen  
INPA Seminar*

## Calculation Approaches:

- Approximate Earth as 'shells' of constant density
- Use constant 'mean' density vs. zenith angle
- Solve Schrodinger equation...

# Earth 'Oscillogram'

## Normal Hierarchy:

Resonant phenomena  
for neutrinos.

'Regular' oscillation  
for anti-neutrinos.

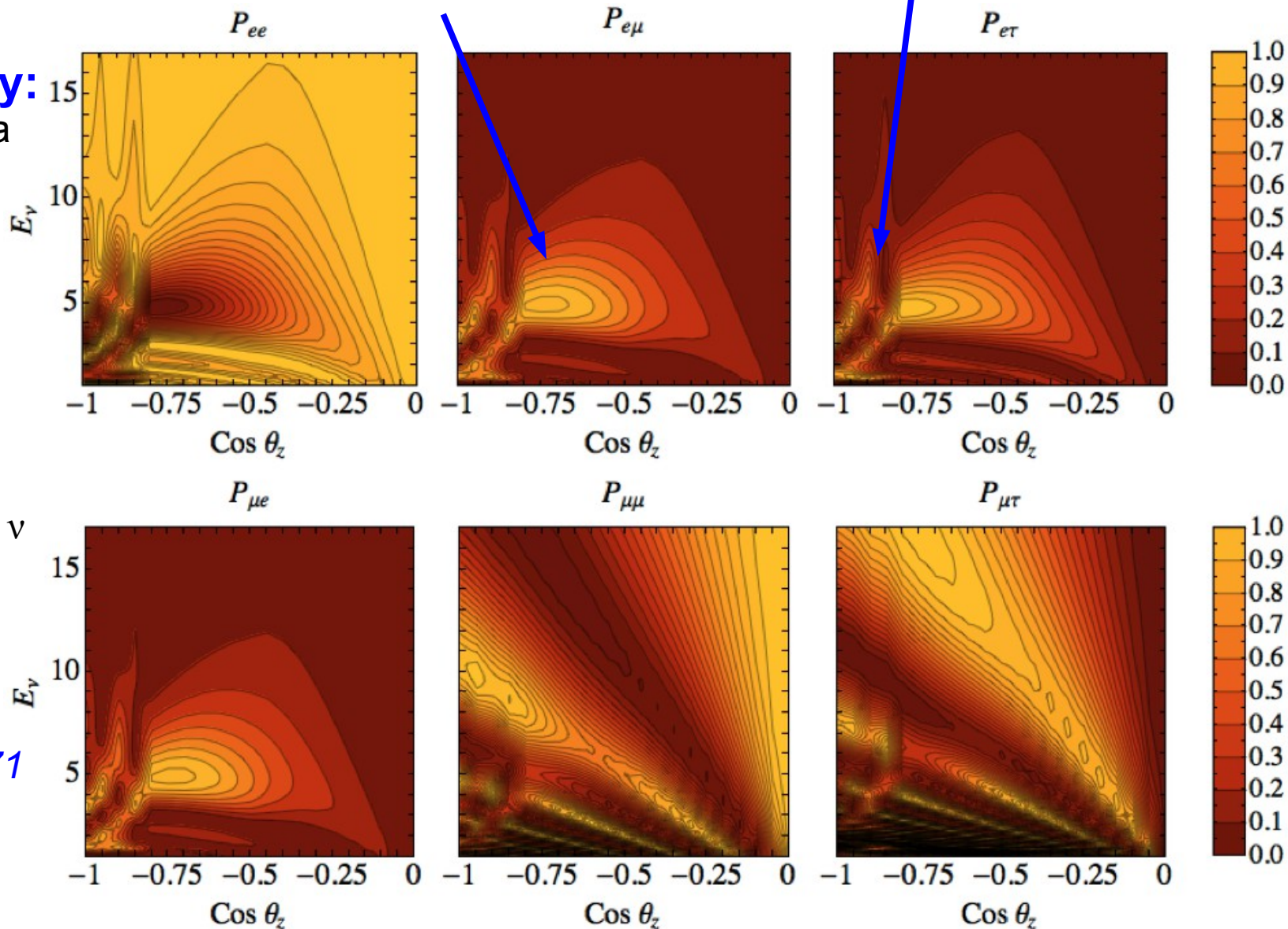
## Atmospheric $\nu$ :

Roughly 2x as many  
muon  $\nu$  vs. anti-muon  $\nu$

From  
[arXiv:1205.7071](https://arxiv.org/abs/1205.7071)

Resonant conversion in mantle

Mantle-core 'effective' resonance



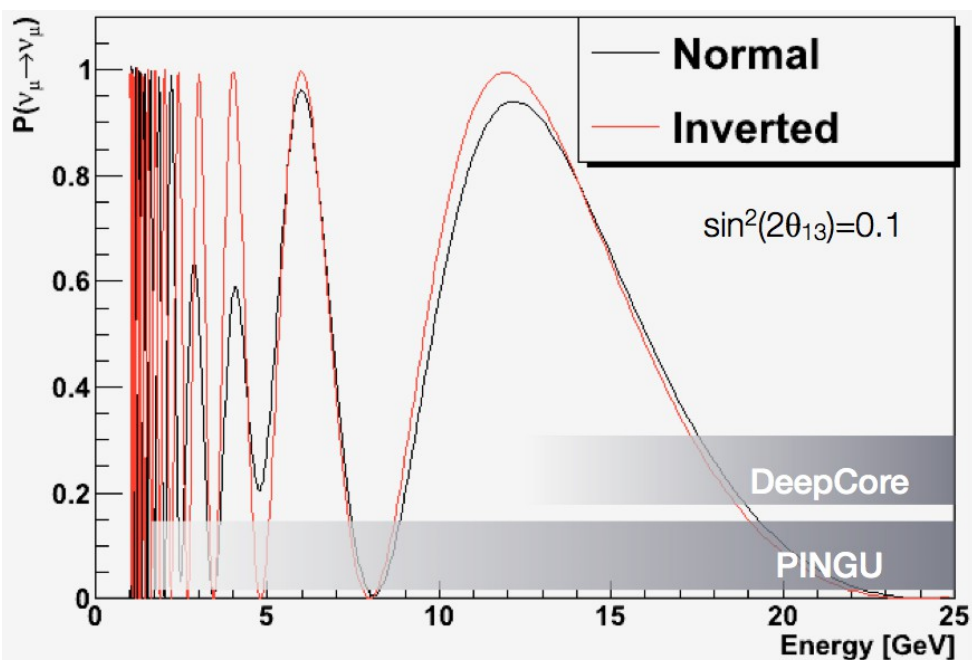
**Inverted Hierarchy:** Matter / anti-matter oscillograms exchanged

# PINGU

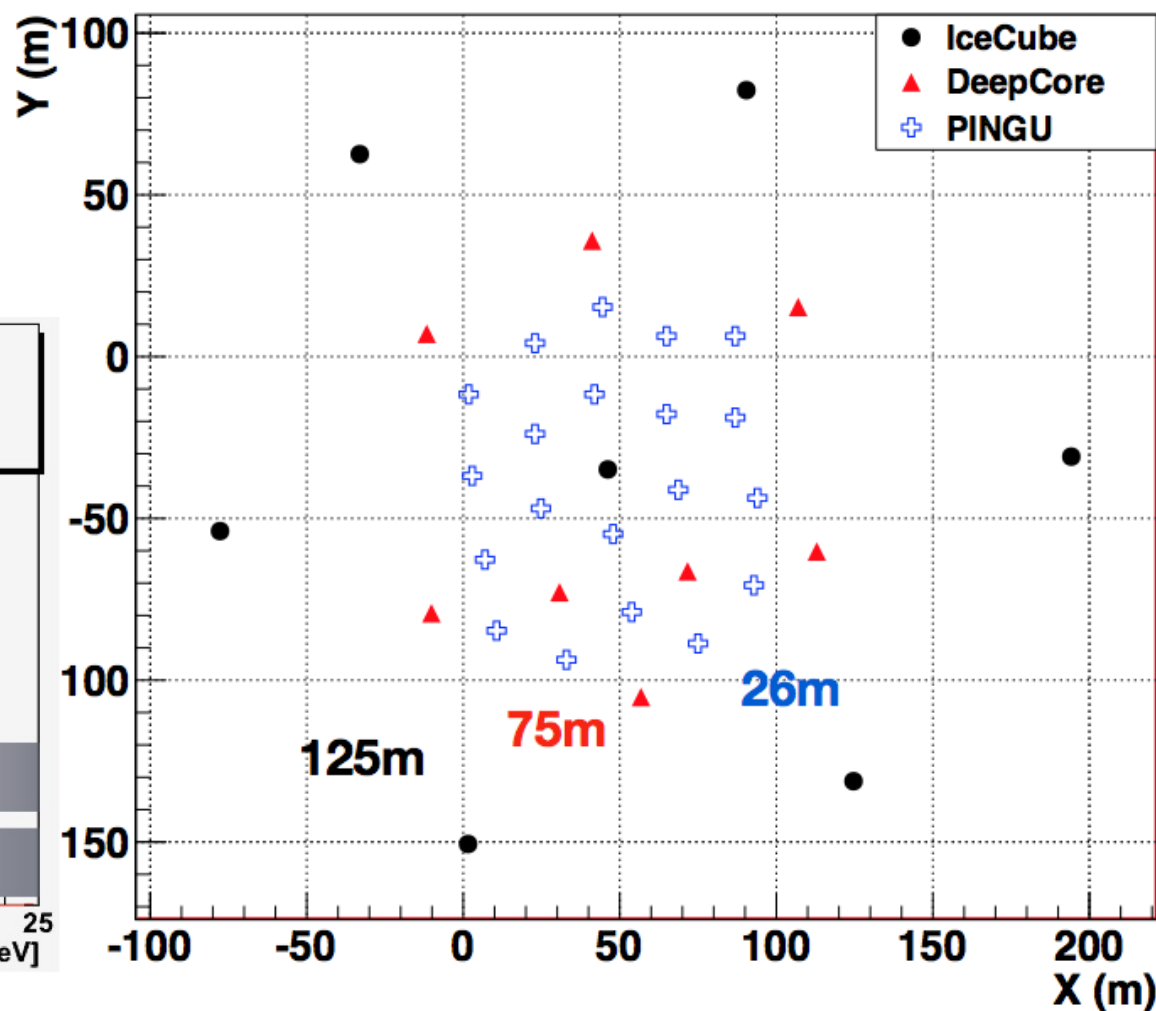
*From J. Koskinen  
INPA Seminar*

## Proposal:

→ Add 20 more 'strings' of PMTs into central region of DeepCore detector.



## PINGU Geometry - 26m String Spacing



# PINGU event rate

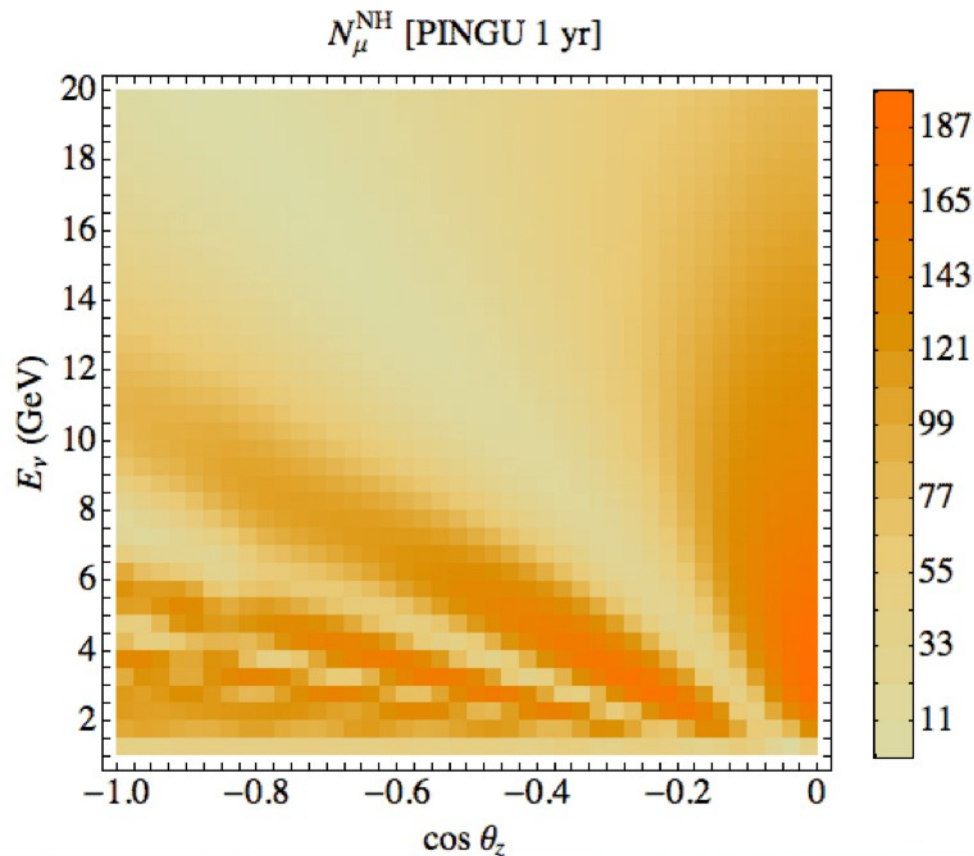
## Calculation Assumptions:

No muon charge discrimination.

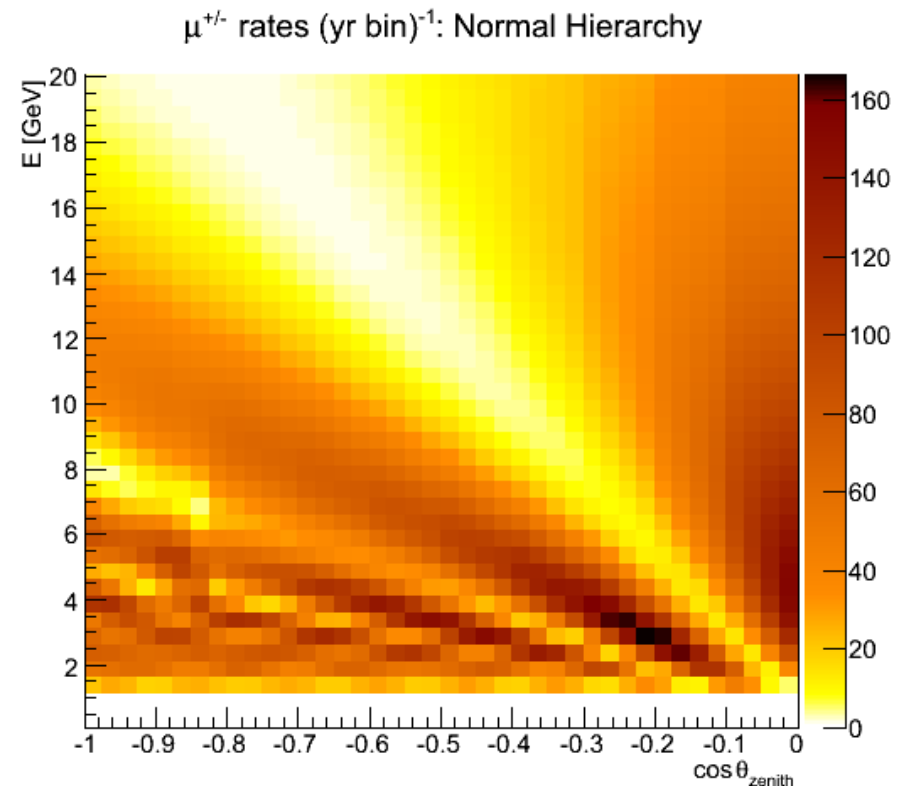
'Perfect' detector performance.

Ignore mis-identified  $\nu_e$  and  $\nu_\tau$  backgrounds.

Energy-dependent fiducial volume (2 Mton at 2 GeV, 20 Mton at 20 GeV)



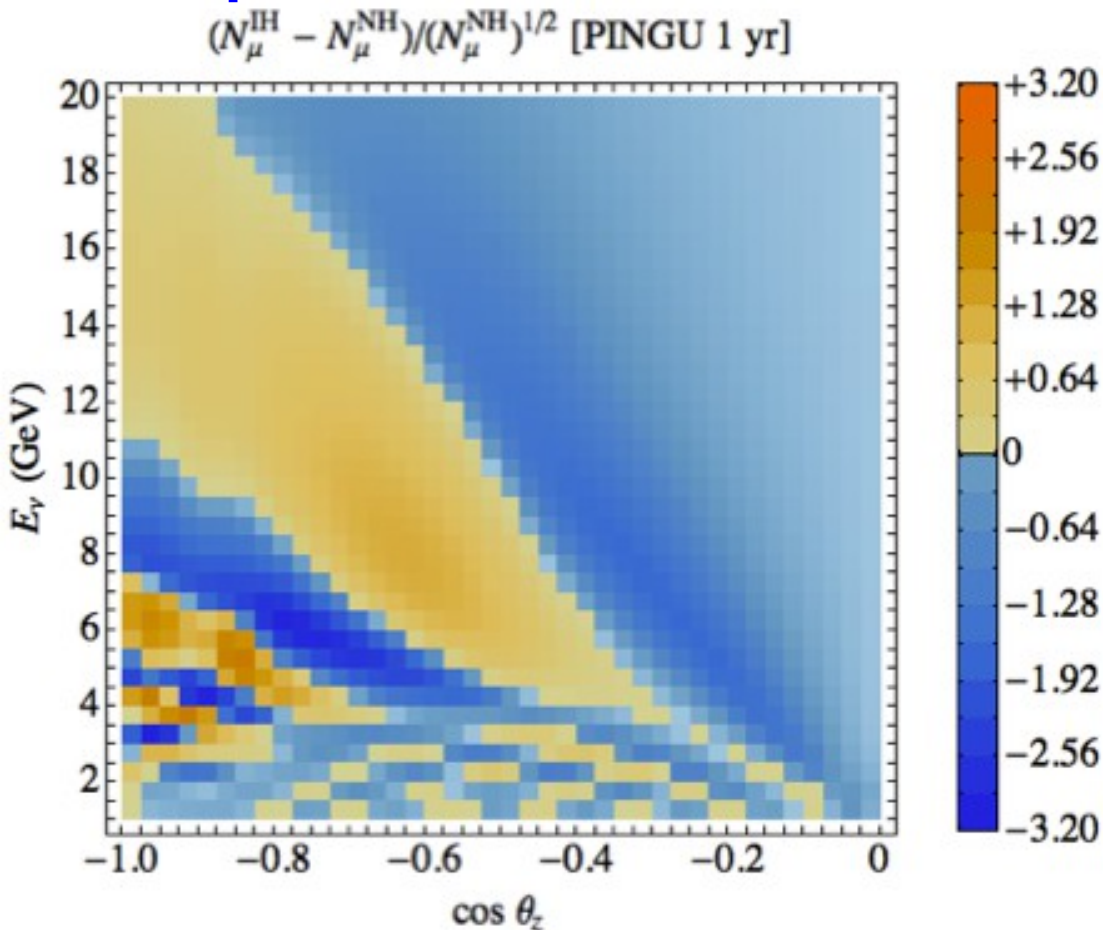
[arXiv:1205.7071](https://arxiv.org/abs/1205.7071)



*My initial calculation confirms event distribution*

# PINGU: Hierarchy discrimination

Compare the muon distribution for normal and inverted hierarchy



[arXiv:1205.7071](https://arxiv.org/abs/1205.7071)

$$\Delta\chi^2 = \sum_i \frac{(NH_{expected}^i - IH_{expected}^i)^2}{(\delta NH_{measured}^i)^2}$$

## Questions:

Not clear how systematics impact the measurement.

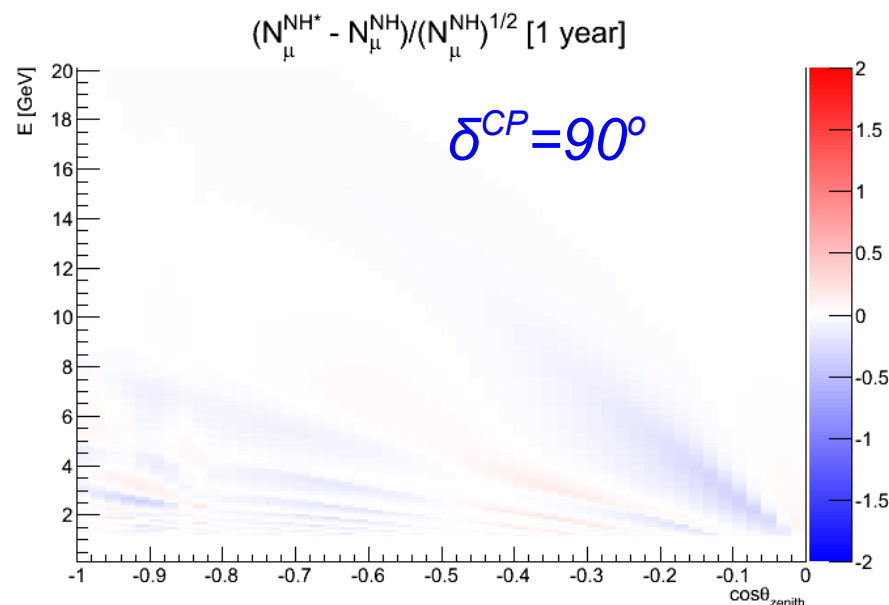
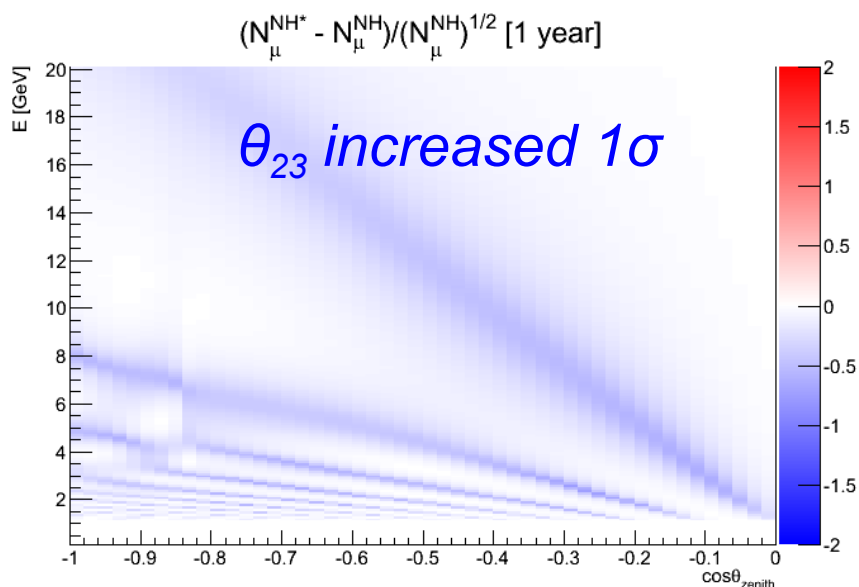
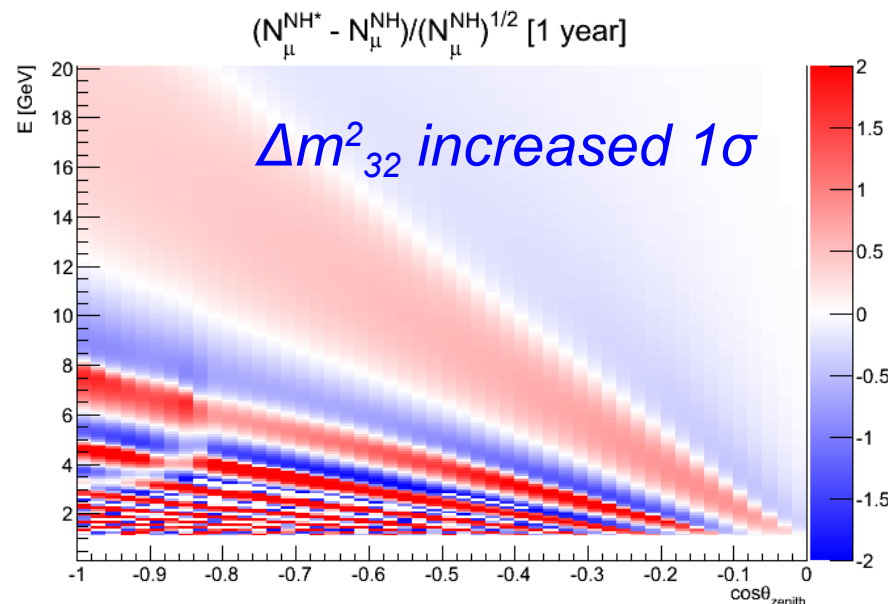
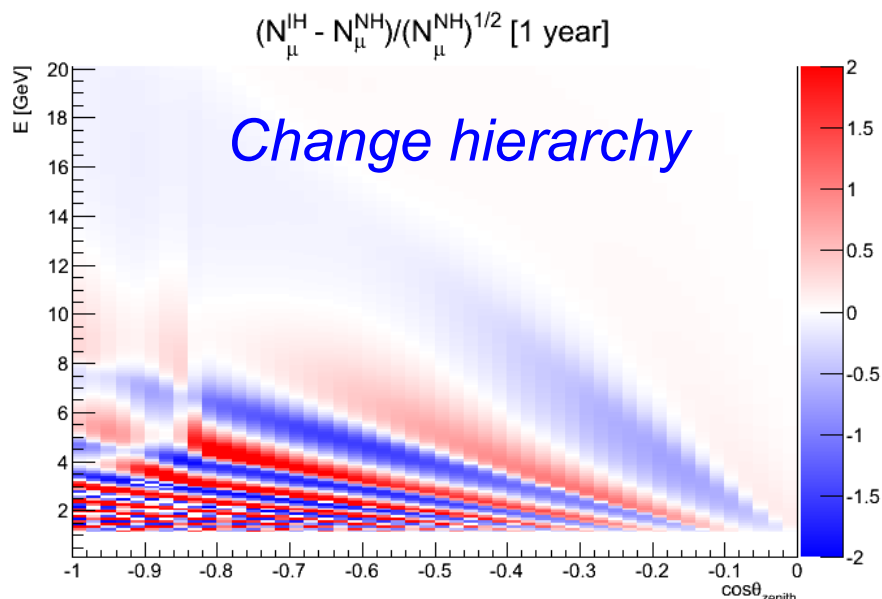
Certain approximations made;  
validity is not clear.

Sensitivity 'optimistic' in current form.

What about 'real' detector performance?

# Oscillation Systematics

*Compare hierarchy change with other modifications to oscillation*

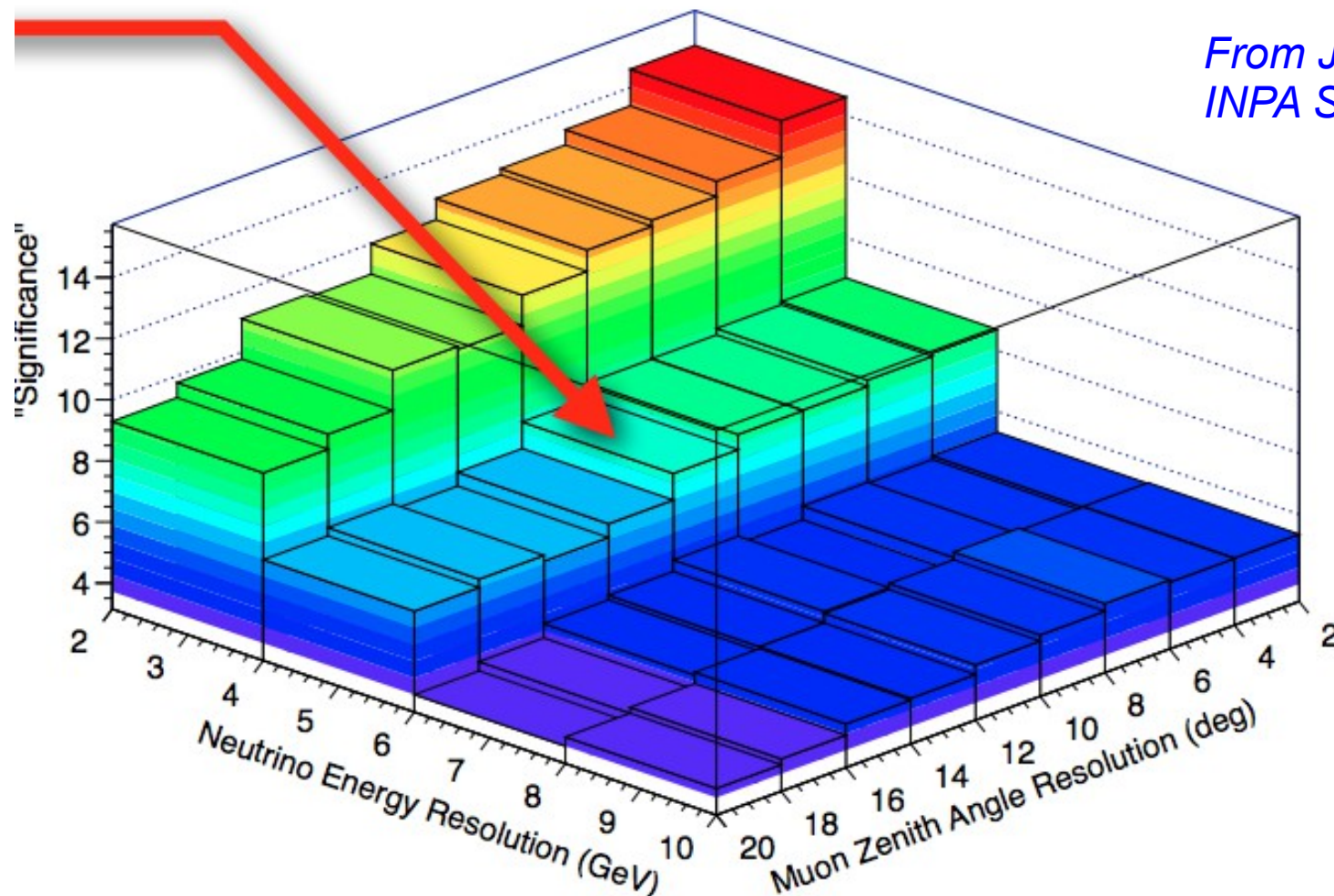


*Systematics significant relative to hierarchy, but have different structure*

# PINGU: Detector Performance

**Study added resolution to distribution, and examined 'significance'.**

Distinguishability PINGU 26m spacing - 1 Year Data Taking, 20 Hit Cut



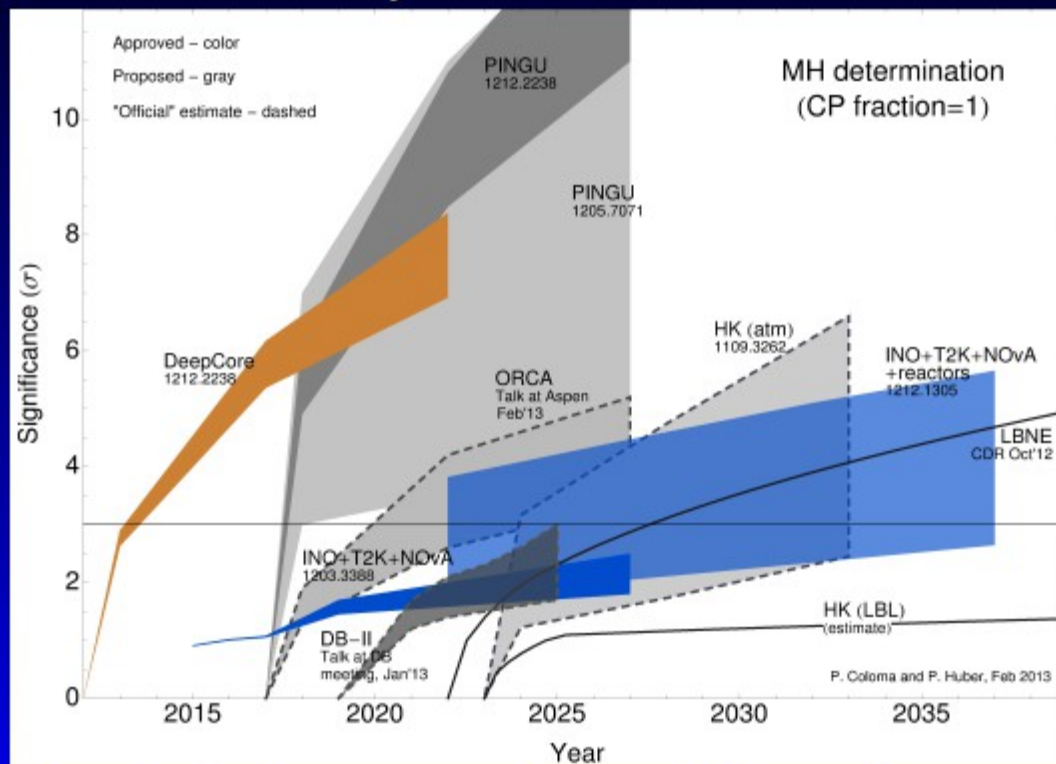
*From J. Koskinen  
INPA Seminar*

**Detailed detector model, complete systematics not included.**

# Sensitivity

## New ideas for mass hierarchy

### Literature survey



The dashed ones are from collaborations –  
phenomenological studies are driving the field.

Patrick Huber, SLAC Intensity Frontier Pre-Snowmass Meeting

# Challenges

## **Comprehensive Sensitivity:**

- Consider all systematics simultaneously:
  - Is intrinsic signal still significant?
  - Backgrounds?

## **Detector Performance:**

- What are detector performance requirements for detection?
- Will PINGU design have sufficient resolution, calibration?

## **Alternative Detectors:**

- ORCA: ??
- INO: too small?
- Hyper-K: Claims  $2-3\sigma$  sensitivity,  $>3\sigma$  when combined with beam

# Summary

## **Atmospheric measurement of neutrino mass hierarchy is unclear:**

- Intrinsic signal is significant:
  - High-statistics for >MT-sized detectors
  - Systematics: Preliminary assessment implies most systematics decoupled from hierarchy
- Current detector designs and sensitivity studies have not made a convincing case:
  - Technology exists to obtain sufficient detector performance (eg. Super-K)
  - Unclear if PINGU or ORCA designs have sufficient energy and angle resolution, calibration.
  - Possible muon charge discrimination would improve sensitivity.

## **LBNL Opportunities:**

### **Comprehensive sensitivity study:**

- A detailed and comprehensive sensitivity study:
  - Provide clearer guidance and detector requirements.
- Explore detector design options:
  - Alternative detection methods
  - For PINGU: Understanding of ice, optics, efficiency, etc.
  - Develop methods to discriminate muon charge

### **Participation in PINGU:**

- Build on existing involvement in IceCUBE.
  - Use expertise to demonstrate feasible detector design and calibration program.